

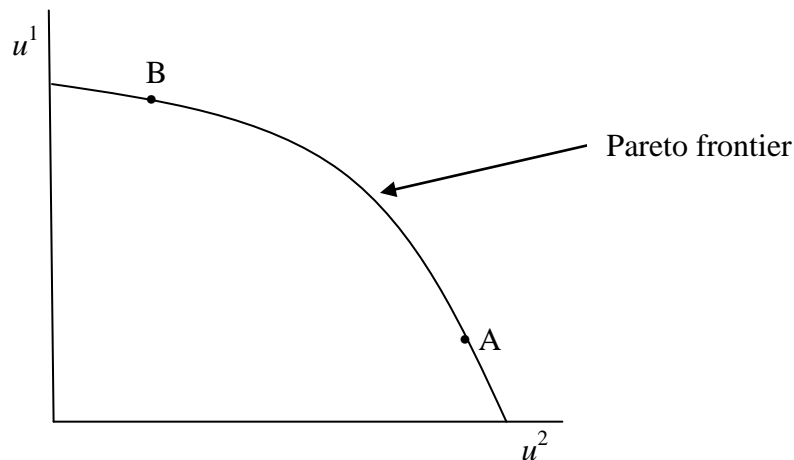
## Economics 230a, Fall 2013

### Lecture Note 1: Welfare Economics and the Role of Government

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Public finance analyzes the role of government in the economy.

To understand this role, start with the two fundamental theorems of welfare economics.



The first fundamental theorem says that, under certain assumptions, all competitive equilibria are Pareto optimal. That is, they lie on the Pareto frontier that defines the set of possible allocations among individuals; on the frontier it is not possible to make someone better off without making someone else worse off.

But Pareto optimality defines optimality in only a limited sense; it does not allow us, for example, to rank outcomes A and B. To do this, we need some mechanism for ranking allocations. We typically use a social welfare function,  $W(u^1, u^2, \dots, u^H)$ . Note that, as the scale of an individual's utility function is arbitrary, so is the social welfare function. All that matters is the combined effect of an increase in individual  $h$ 's income, e.g.,  $W_h \partial u^h / \partial y^h$ . Also, we assume that we can make interpersonal comparisons, even among individuals with different tastes (perhaps by normalizing so that they get the same welfare weights at the same levels of income).

Note that we typically assume that  $W_h \geq 0$ ; that is, the social welfare function is non-decreasing in individual well-being and therefore will achieve a maximum at some Pareto optimum. Note also that the social welfare function is based only on individual well-being, and therefore does not override individual preferences or incorporate other measures, for inequality, for example.

The second fundamental theorem says that each Pareto optimum can be achieved via a competitive equilibrium, if lump-sum taxes and transfers are available to shift individual endowments. For example, if initial endowments yield point A and our social welfare function prefers point B, we can impose a lump-sum tax on individual 2 and give it to individual 1 to induce this shift in the resulting equilibrium.

Based on the fundamental theorems, we have established a role for government, but it is a very limited one: to impose lump-sum taxes and transfers. This is quite removed from the government activity we observe, so what is missing?

First, government's ability to use lump-sum taxes to improve the distribution of resources may be limited, and this leads to the use of more realistic taxes and transfer payments.

Second, the analysis so far does not take account of market failures, which can result for many reasons. If market failures exist, then a competitive equilibrium will generally not be Pareto optimal, so government intervention in the form of government spending, non-lump-sum taxes, and regulations, may improve outcomes.

## Important Market Failures

The two classic types of market failures are public goods and externalities.

“Pure” public goods are defined as having two key characteristics:

1. Nonrival in consumption:  $x^1 = x^2 = \dots = x^H = x$ .
2. Nonexcludable: no individual can be kept from consuming all of  $x$  if it is produced.

Characteristic 1 means that we want everyone to consume the good, because it is costless to do so. Characteristic 2 means that private provision, even inefficient provision in which individuals have to pay to access the commodity, is not feasible, since individuals cannot be excluded from consuming and therefore will choose to pay 0.

If both conditions are satisfied, only public provision (or publicly funded private provision) is possible. If only condition 1 is satisfied, then private provision is possible (example – software) but will not be efficient.

Optimal provision:  $\max W(u^1, u^2, \dots, u^H) \quad \text{s.t.} \quad F(\mathbf{X}, G) \leq 0$ , where  $u^h = u^h(\mathbf{x}^h, G)$ ,  $\sum_h \mathbf{x}^h = \mathbf{X}$ .  $F(\cdot)$  is a convex, constant returns to scale production function (homogeneous of degree zero), where inputs are negative and outputs are positive;  $\mathbf{X}$  is the vector of private inputs and outputs and  $G$  is the output of the public good.

Form a Lagrangian  $L = W(u^1, u^2, \dots, u^H) - \mu F(\mathbf{X}, G)$ ; first order conditions are:

$$x_i^h: \quad W^h u_i^h = \mu F_i \quad \forall h, i$$

$$G: \quad \sum_h W^h u_G^h = \mu G$$

Combining the first condition for different  $i$  and  $h$  yields the standard result that  $MRS = MRT$  for all goods and all individuals. Dividing the second condition by the first (ranging over  $h$ ) yields:

$$\sum_h \frac{u_G^h}{u_i^h} = \frac{F_G}{F_i}$$

This says that we should sum  $MRS^h$  and set equal to  $MRT$ , because everyone consumes the public good. (This is sometimes referred to as vertical summation of demand curves.) This result is due to Samuelson (1954).

Problem: if we don't have a market, how do we know individual valuations? This lack of information explains why we might settle for private provision (in the case of excludability), even if it falls short of Pareto optimality.

Externalities represent a market failure or market absence that is associated with a functioning market. For example, pollution may result from production in a market that is competitive, but there is no market for the pollution itself. There are many ways to represent externalities, but consider an "atmospheric externality" to which all contribute and which affects all. That is, individual utility is  $u^h(x^h, X_N)$ , where  $X_N$  is aggregate output of the  $N$ th consumption good. It can have a positive or negative effect on utility, corresponding to positive and negative externalities.

Assuming a CRS production function  $F(X)$  and forming a Lagrangian, we get the first order conditions:

$$x_i^h: W^h u_i^h = \mu F_i \quad \forall h, i \neq N$$

$$x_N^h: W^h u_N^h + \sum_h W^h u_{N+1}^h = \mu F_N \quad \forall h$$

The second condition includes an extra term to account for the impact that individual  $h$ 's consumption has on all others. Dividing the second condition by the first (ranging over  $h$ ) yields:

$$\frac{u_N^h}{u_i^h} = \frac{F_N}{F_i} - \sum_h \frac{u_{N+1}^h}{u_i^h} \quad \forall h$$

How can we achieve this outcome? In theory, we can do so by imposing a Pigouvian tax (subsidy) on each individual, equal to the damage (benefit) that individual's consumption of good  $N$  causes others. Again, though, we must know the damage or benefit in order to do so.

Other sources of market failure include imperfect competition and imperfect information. One may also include in this category so-called merit goods – cases where we may wish to override individual decisions for reasons of paternalism or because individual choices for some reason (other than imperfect information) fail to reflect the individuals' underlying preferences.